

Formulario de Precálculo.

1. Los Números.

1. Leyes de los exponentes y radicales.

a) $a^m a^n = a^{m+n}$ b) $(a^m)^n = a^{mn}$ c) $(ab)^n = a^n b^n$
d) $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ e) $\frac{a^m}{a^n} = a^{m-n}$ f) $a^{-n} = \frac{1}{a^n}$
g) $a^{1/n} = \sqrt[n]{a}$ h) $a^{m/n} = \sqrt[n]{a^m}$ i) $a^{m/n} = (\sqrt[n]{a})^m$
j) $\sqrt[n]{ab} = \sqrt[n]{a} \sqrt[n]{b}$ k) $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$ l) $\sqrt[n]{\sqrt[n]{a}} = \sqrt[n^2]{a}$

2. Productos Notables.

a) Binomios Conjugados: $(x+y)(x-y) = x^2 - y^2$
b) Binomio al Cuadrado: $(x \pm y)^2 = x^2 \pm 2xy + y^2$
c) Binomio al Cubo: $(x \pm y)^3 = x^3 \pm 3x^2y + 3xy^2 \pm y^3$
d) $(x+y)^2 = x^2 + 2xy + y^2$
e) $(x-y)^2 = x^2 - 2xy + y^2$
f) $(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$
g) $(x-y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$
h) $(x+y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$
i) $(x-y)^4 = x^4 - 4x^3y + 6x^2y^2 - 4xy^3 + y^4$
j) $(x+y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$
k) $(x-y)^5 = x^5 - 5x^4y + 10x^3y^2 - 10x^2y^3 + 5xy^4 - y^5$

3. Teorema del Binomio. Sea $n \in \mathbb{N}$, entonces:

$$(x+y)^n = \sum_{r=0}^n \binom{n}{r} x^{n-r} y^r$$

Nota: $\binom{n}{r} = {}_n C_r = \frac{n!}{r!(n-r)!}$

4. Factores Notables.

a) Diferencia de Cuadrados: $x^2 - y^2 = (x+y)(x-y)$
b) Suma de Cubos: $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$
c) Diferencia de Cubos: $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$
d) Trinomio Cuadrado Perfecto: $x^2 \pm 2xy + y^2 = (x \pm y)^2$
e) $x^2 - y^2 = (x-y)(x+y)$
f) $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$
g) $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$
h) $x^4 - y^4 = (x-y)(x+y)(x^2 + y^2)$
i) $x^5 - y^5 = (x-y)(x^4 + x^3y + x^2y^2 + xy^3 + y^4)$
j) $x^5 + y^5 = (x+y)(x^4 - x^3y + x^2y^2 - xy^3 + y^4)$
k) $x^6 - y^6 = (x-y)(x+y)(x^2 + xy + y^2)(x^2 - xy + y^2)$
l) $x^4 + x^2y^2 + y^4 = (x^2 + xy + y^2)(x^2 - xy + y^2)$
m) $x^4 + 4y^4 = (x^2 - 2xy + 2y^2)(x^2 + 2xy + 2y^2)$

5. Leyes de los logaritmos.

a) $\log_a(PQ) = \log_a(P) + \log_a(Q)$
b) $\log_a\left(\frac{P}{Q}\right) = \log_a(P) - \log_a(Q)$
c) $\log_a(Q^n) = n \log_a(Q)$
d) $a^{\log_a(x)} = x$
e) $\log_a(a^x) = x$
f) $\log_a(1) = 0$
g) $a^{\log_a(a)} = 1$
h) $\log(x) = \log_{10}(x)$
i) $\ln(x) = \log_e(x)$
j) Cambio de base: $\log_a(Q) = \frac{\log_b(Q)}{\log_b(a)}$

2. Soluciones Exactas de ecuaciones Algebraicas

6. Soluciones Exactas de Ecuaciones Algebraicas.

a) La **Ecuación Cuadrática**: $ax^2 + bx + c = 0$ tiene soluciones:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

El número $b^2 - 4ac$ se llama *discriminante* de la ecuación.

- i) Si $b^2 - 4ac > 0$ las raíces son reales y diferentes.
- ii) Si $b^2 - 4ac = 0$ las raíces son reales e iguales.
- iii) Si $b^2 - 4ac < 0$ las raíces son complejas conjugadas.

b) Para la **Ecuación Cúbica**: $x^3 + ax^2 + bx + c = 0$ sean:

$$Q = \frac{3b - a^2}{9}, \quad R = \frac{9ab - 27c - 2a^3}{54}$$

$$S = \sqrt[3]{R + \sqrt{Q^3 + R^2}}, \quad T = \sqrt[3]{R - \sqrt{Q^3 + R^2}}$$

Entonces las soluciones son:

$$\begin{aligned} x_1 &= S + T - \frac{a}{3} \\ x_2 &= -\left(\frac{S+T}{2} + \frac{a}{3}\right) + \left(\frac{(S-T)\sqrt{3}}{2}\right)i \\ x_3 &= -\left(\frac{S+T}{2} + \frac{a}{3}\right) - \left(\frac{(S-T)\sqrt{3}}{2}\right)i \end{aligned}$$

El número $Q^3 + R^2$ se llama *discriminante* de la ecuación.

- i) Si $Q^3 + R^2 > 0$, hay una raíz real y dos son complejas conjugadas.
- ii) Si $Q^3 + R^2 = 0$, las raíces son reales y por lo menos dos son iguales.
- iii) Si $Q^3 + R^2 < 0$, las raíces son reales y diferentes.

3. Funciones Trigonométricas.

3.1. Relaciones entre Funciones Trigonométricas.

$\csc(A) = \frac{1}{\sin(A)}$	$\sin^2(A) + \cos^2(A) = 1$
$\sec(A) = \frac{1}{\cos(A)}$	$\sec^2(A) - \tan^2(A) = 1$
$\tan(A) = \frac{\sin(A)}{\cos(A)}$	$\csc^2(A) - \cot^2(A) = 1$
$\cot(A) = \frac{\cos(A)}{\sin(A)} = \frac{1}{\tan(A)}$	

3.2. Potencias de Funciones Trigonométricas.

$$\sin^2(A) = \frac{1}{2} - \frac{1}{2} \cos(2A)$$

$$\cos^2(A) = \frac{1}{2} + \frac{1}{2} \cos(2A)$$

$$\sin^3(A) = \frac{3}{4} \sin(A) - \frac{1}{4} \sin(3A)$$

4. Funciones Hiperbólicas.

$$\text{Seno hiperbólico de } x = \sinh(x) = \frac{e^x - e^{-x}}{2}$$

$$\text{Coseno hiperbólico de } x = \cosh(x) = \frac{e^x + e^{-x}}{2}$$

$$\text{Tangente hiperbólica de } x = \tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

4.1. Relación entre las Funciones Hiperbólicas.

$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)}$$

$$\operatorname{sech}(x) = \frac{1}{\cosh(x)}$$

$$\coth(x) = \frac{1}{\tanh(x)} = \frac{\cosh(x)}{\sinh(x)}$$

$$\operatorname{csch}(x) = \frac{1}{\sinh(x)}$$

$$\cosh^2(x) - \sinh^2(x) = 1$$

$$\operatorname{sech}^2(x) + \tanh^2(x) = 1$$

$$\coth^2(x) - \operatorname{csch}^2(x) = 1$$

$$\cos^3(A) = \frac{3}{4} \cos(A) + \frac{1}{4} \cos(3A)$$

$$\sin^4(A) = \frac{3}{8} - \frac{1}{2} \cos(2A) + \frac{1}{8} \cos(4A)$$

$$\cos^4(A) = \frac{3}{8} + \frac{1}{2} \cos(2A) + \frac{1}{8} \cos(4A)$$

$$\sin^5(A) = \frac{5}{8} \sin(A) - \frac{5}{16} \sin(3A) + \frac{1}{16} \sin(5A)$$

$$\cos^5(A) = \frac{5}{8} \cos(A) + \frac{5}{16} \cos(3A) + \frac{1}{16} \cos(5A)$$

3.3. Suma, Diferencia y Producto las Funciones Trigonométricas.

$$\sin(A) + \sin(B) = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$$

$$\sin(A) - \sin(B) = 2 \sin\left(\frac{A-B}{2}\right) \cos\left(\frac{A+B}{2}\right)$$

$$\cos(A) + \cos(B) = 2 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$$

$$\cos(A) - \cos(B) = 2 \sin\left(\frac{A+B}{2}\right) \sin\left(\frac{B-A}{2}\right)$$

$$\sin(A) \sin(B) = \frac{1}{2} [\cos(A-B) - \cos(A+B)]$$

$$\cos(A) \cos(B) = \frac{1}{2} [\cos(A-B) + \cos(A+B)]$$

$$\sin(A) \cos(B) = \frac{1}{2} [\sin(A-B) + \sin(A+B)]$$

$$\text{Cosecante hiperbólica de } x = \operatorname{csch}(x) = \frac{2}{e^x - e^{-x}}$$

$$\text{Secante hiperbólica de } x = \operatorname{sech}(x) = \frac{2}{e^x + e^{-x}}$$

$$\text{Cotangente hiperbólica de } x = \coth(x) = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

Formulario de Cálculo.

Derivadas.

En este formulario: $k, c \in \mathbb{R}$ son constantes reales, $f = f(x)$, $u = u(x)$ y $v = v(x)$ son funciones que dependen de x .

Fórmulas Básicas:

Función: Su Derivada:

$$f = k \quad f' = 0$$

Linealidad de la derivada:

$$f = k \cdot u \quad f' = k \cdot u'$$

$$f = u \pm v \quad f' = u' \pm v'$$

$$f = k \cdot u \pm c \cdot v \quad f' = k \cdot u' \pm c \cdot v'$$

Regla del Producto:

$$f = u \cdot v \quad f' = u \cdot v' + v \cdot u'$$

Regla del Cociente:

$$f = \frac{u}{v} \quad f' = \frac{v \cdot u' - u \cdot v'}{v^2}$$

Regla de la Cadena (Composición de funciones)

$$f = u(x) \circ v(x) \quad f' = [u(v(x))]' \cdot v'(x)$$

Regla de la Potencia:

$$f = v^n \quad f' = n \cdot v^{n-1} \cdot v'$$

$$f = k \cdot v^n \quad f' = k \cdot n \cdot v^{n-1} \cdot v'$$

Funciones Exponenciales:

$$f = e^u \quad f' = e^u \cdot u'$$

$$f = a^u \quad f' = a^u \cdot \ln(a) \cdot u'$$

Funciones Logarítmicas:

$$f = \ln(u) \quad f' = \frac{u'}{u}$$

$$f = \log_a(u) \quad f' = \frac{u'}{u \cdot \ln(a)}$$

Una Función elevada a otra Función:

$$f = u^v \quad f' = u^v \left[v' \cdot \ln(u) + \frac{v \cdot u'}{u} \right]$$

Funciones Trigonométricas:

Función: Su Derivada:

$$f = \text{sen}(u) \quad f' = \cos(u) \cdot u'$$

$$f = \cos(u) \quad f' = -\text{sen}(u) \cdot u'$$

$$f = \tan(u) \quad f' = \sec^2(u) \cdot u'$$

$$f = \csc(u) \quad f' = -\csc(u) \cot(u) \cdot u'$$

$$f = \sec(u) \quad f' = \sec(u) \tan(u) \cdot u'$$

$$f = \cot(u) \quad f' = -\csc^2(u) \cdot u'$$

Funciones Trigonométricas Inversas:

Función: Su Derivada:

$$f = \arcsen(u) \quad f' = \frac{u'}{\sqrt{1-u^2}}; \quad |u| < 1$$

$$f = \arccos(u) \quad f' = -\frac{u'}{\sqrt{1-u^2}}; \quad |u| < 1$$

$$f = \arctan(u) \quad f' = \frac{u'}{1+u^2}$$

$$f = \text{arccsc}(u) \quad f' = -\frac{u'}{u\sqrt{u^2-1}}$$

$$f = \text{arcsec}(u) \quad f' = \frac{u'}{u\sqrt{u^2-1}}; \quad |u| > 1$$

$$f = \text{arccot}(u) \quad f' = -\frac{u'}{1+u^2}; \quad |u| > 1$$

Funciones Hiperbólicas:

Función: Su Derivada:

$$f = \text{senh}(u) \quad f' = \cosh(u) \cdot u'$$

$$f = \cosh(u) \quad f' = \text{senh}(u) \cdot u'$$

$$f = \tanh(u) \quad f' = \text{sech}^2(u) \cdot u'$$

$$f = \text{csch}(u) \quad f' = -\text{csch}(u) \coth(u) \cdot u'$$

$$f = \text{sech}(u) \quad f' = -\text{sech}(u) \tanh(u) \cdot u'$$

$$f = \coth(u) \quad f' = -\text{csch}^2(u) \cdot u'$$

Funciones Hiperbólicas Inversas:

Función: Su Derivada:

$$f = \operatorname{arcsenh}(u) \quad f' = \frac{u'}{\sqrt{1+u^2}}$$

$$f = \operatorname{arccosh}(u) \quad f' = \frac{u'}{\sqrt{u^2-1}}; \quad |u| > 1$$

$$f = \operatorname{arctanh}(u) \quad f' = \frac{u'}{1-u^2}; \quad |u| < 1$$

$$f = \operatorname{arccsch}(u) \quad f' = -\frac{u'}{|u|\sqrt{1+u^2}}; \quad u \neq 0$$

$$f = \operatorname{arcsech}(u) \quad f' = -\frac{u'}{u\sqrt{1-u^2}}; \quad 0 < u < 1$$

$$f = \operatorname{arccoth}(u) \quad f' = \frac{u'}{1-u^2}; \quad |u| > 1$$

Integrales.

En este formulario: $k, w, C \in \mathbb{R}$ son constantes reales, $u = u(x)$ y $v = v(x)$ son funciones que dependen de x .

Fórmulas Básicas.

- 1) $\int 0 dx = C$
- 2) $\int k dx = kx + C$
- 3) $\int (k \cdot u \pm w \cdot v) dx = k \int u dx + w \int v dx + C$
- 4) Regla de la potencia $\int u^n du = \frac{u^{n+1}}{n+1}$ para $n \neq -1$.
- 5) Regla exponencial $\int e^u du = e^u$
- 6) Regla logarítmica $\int \ln |u| du = u \ln |u| - u$
- 7) $\int a^u du = \frac{a^u}{\ln(a)} + C$
- 8) $\int \frac{du}{u} = \ln |u| + C$

Trigonométricas.

- 9) $\int \sin u du = -\cos u$
- 10) $\int \cos u du = \sin u$
- 11) $\int \tan u du = \ln|\sec u| = -\ln|\cos u| + C$
- 12) $\int \cot u du = \ln|\sin u|$
- 13) $\int \sec u du = \ln|\sec u + \tan u| = \ln\left|\tan\left(\frac{u}{2} + \frac{\pi}{4}\right)\right|$
- 14) $\int \csc u du = \ln|\csc u - \cot u| = \ln\left|\tan\frac{u}{2}\right|$
- 15) $\int \sec^2 u du = \tan u$
- 16) $\int \csc^2 u du = -\cot u$

- 17) $\int \tan^2 u du = \tan u - u$
- 18) $\int \cot^2 u du = -\cot u - u$
- 19) $\int \sin^2 u du = \frac{u}{2} - \frac{\sin 2u}{4} = \frac{1}{2}[u - \sin u \cos u]$
- 20) $\int \cos^2 u du = \frac{u}{2} + \frac{\sin 2u}{4} = \frac{1}{2}[u + \sin u \cos u]$
- 21) $\int \sec u \tan u du = \sec u$
- 22) $\int \csc u \cot u du = -\csc u$

Hiperbólicas.

- 23) $\int \sinh u du = \cosh u$
- 24) $\int \cosh u du = \sinh u$
- 25) $\int \tanh u du = \ln|\cosh u|$
- 26) $\int \coth u du = \ln|\sinh u|$
- 27) $\int \operatorname{sech} u du = \sin^{-1}[\tanh u] = 2 \tan^{-1}[e^u]$
- 28) $\int \operatorname{csch} u du = \ln\left|\tanh \frac{u}{2}\right| = -2 \coth^{-1}[e^u]$
- 29) $\int \operatorname{sech}^2 u du = \tanh u$
- 30) $\int \operatorname{csch}^2 u du = -\coth u$
- 31) $\int \tanh^2 u du = u - \tanh u$
- 32) $\int \coth^2 u du = u - \coth u$
- 33) $\int \sinh^2 u du = \frac{\sinh 2u}{4} - \frac{u}{2} = \frac{1}{2}[\sinh u \cosh u - u]$
- 34) $\int \cosh^2 u du = \frac{\sinh 2u}{4} + \frac{u}{2} = \frac{1}{2}[\sinh u \cosh u + u]$
- 35) $\int \operatorname{sech} u \tanh u du = -\operatorname{sech} u$
- 36) $\int \operatorname{csch} u \coth u du = -\operatorname{csch} u$

Integrales con $au + b$.

- 37) $\int \frac{du}{au+b} = \frac{1}{a} \ln(au+b)$
- 38) $\int \frac{u du}{au+b} = \frac{u}{a} - \frac{b}{a^2} \ln(au+b)$
- 39) $\int \frac{u^2 du}{au+b} = \frac{(au+b)^2}{2a^3} - \frac{2b(au+b)}{a^3} + \frac{b^2}{a^3} \ln(au+b)$
- 40) $\int \frac{u^3 du}{au+b} = \frac{(au+b)^3}{3a^4} - \frac{3b(au+b)^2}{2a^4} + \frac{3b^2(au+b)}{a^4} - \frac{b^3}{a^4} \ln(au+b)$
- 41) $\int \frac{du}{u(au+b)} = \frac{1}{b} \ln\left(\frac{u}{au+b}\right)$
- 42) $\int \frac{du}{u^2(au+b)} = -\frac{1}{bu} + \frac{a}{b^2} \ln\left(\frac{au+b}{u}\right)$
- 43) $\int \frac{du}{(au+b)^2} = \frac{-1}{a(au+b)}$
- 44) $\int \frac{u du}{(au+b)^2} = \frac{b}{a^2(au+b)} + \frac{1}{a^2} \ln(au+b)$
- 45) $\int \frac{u^2 du}{(au+b)^2} = \frac{au+b}{a^3} - \frac{b^2}{a^3(au+b)} - \frac{2b}{a^3} \ln(au+b)$
- 46) $\int \frac{du}{u(au+b)^2} = \frac{1}{b(au+b)} + \frac{1}{b^2} \ln\left(\frac{u}{au+b}\right)$
- 47) $\int \frac{du}{u^2(au+b)^2} = \frac{-a}{b^2(au+b)} - \frac{1}{b^2 u} + \frac{2a}{b^3} \ln\left(\frac{au+b}{u}\right)$
- 48) $\int \frac{du}{(au+b)^3} = \frac{-1}{2(au+b)^2}$

$$49) \int \frac{u du}{(au+b)^3} = \frac{-1}{a^2(au+b)} + \frac{b}{2a^2(au+b)^2}$$

$$50) \int \frac{u^2 du}{(au+b)^3} = \frac{2b}{a^3(au+b)} - \frac{b^2}{2a^3(au+b)^2} + \frac{1}{a^3} \ln(au+b)$$

$$51) \int (au+b) du = \frac{(au+b)^2}{2a}$$

$$52) \int (au+b)^n du = \frac{(au+b)^{n+1}}{(n+1)a} \quad \text{para } n \neq -1$$

$$53) \int u(au+b)^n du = \frac{(au+b)^{n+2}}{(n+2)a^2} - \frac{b(au+b)^{n+1}}{(n+1)a^2} \quad \text{para } n \neq -1, -2$$

$$54) \int u^2(au+b)^n du = \frac{(au+b)^{n+3}}{(n+3)a^3} - \frac{2b(au+b)^{n+2}}{(n+2)a^3} + \frac{b^2(au+b)^{n+1}}{(n+1)a^3}$$

para $n \neq -1, -2, -3$

$$55) \int u^m(au+b)^n du =$$

$$= \begin{cases} \frac{u^{m+1}(au+b)^n}{m+n+1} + \frac{nb}{m+n+1} \int u^m(au+b)^{n-1} du \\ \frac{u^m(au+b)^{n+1}}{(m+n+1)a} - \frac{mb}{(m+n+1)a} \int u^{m-1}(au+b)^n du \\ \frac{-u^{m+1}(au+b)^{n+1}}{(n+1)b} + \frac{m+n+2}{(n+1)b} \int u^m(au+b)^{n+1} du \end{cases}$$

Integrales con $\sqrt{au+b}$.

$$56) \int \frac{du}{\sqrt{au+b}} = \frac{2\sqrt{au+b}}{a}$$

$$57) \int \frac{u du}{\sqrt{au+b}} = \frac{2(au-2b)}{3a^2} \sqrt{au+b}$$

$$58) \int \frac{u^2 du}{\sqrt{au+b}} = \frac{2(3a^2 u^2 - 4ab u + 8b^2)}{15a^3} \sqrt{au+b}$$

$$59) \int \frac{du}{u\sqrt{au+b}} = \begin{cases} \frac{1}{\sqrt{b}} \ln \left(\frac{\sqrt{au+b}-\sqrt{b}}{\sqrt{au+b}+\sqrt{b}} \right) \\ \frac{2}{\sqrt{-b}} \tan^{-1} \sqrt{\frac{au+b}{-b}} \end{cases}$$

$$60) \int \frac{du}{u^2 \sqrt{au+b}} = -\frac{\sqrt{au+b}}{bu} - \frac{a}{2b} \int \frac{du}{u \sqrt{au+b}}$$

$$61) \int \sqrt{au+b} du = \frac{2\sqrt{(au+b)^3}}{3a}$$

$$62) \int u \sqrt{au+b} du = \frac{2(3au-2b)}{15a^2} \sqrt{(au+b)^3}$$

$$63) \int u^2 \sqrt{au+b} du = \frac{2(15a^2 u^2 - 12ab u + 8b^2)}{105a^3} \sqrt{(au+b)^3}$$

$$64) \int \frac{\sqrt{au+b}}{u} du = 2\sqrt{au+b} + b \int \frac{du}{u\sqrt{au+b}}$$

$$65) \int \frac{\sqrt{au+b}}{u^2} du = -\frac{\sqrt{au+b}}{u} + \frac{a}{2} \int \frac{du}{u\sqrt{au+b}}$$

$$66) \int \frac{u^m}{\sqrt{au+b}} du = \frac{2u^m \sqrt{au+b}}{(2m+1)a} - \frac{2mb}{(2m+1)a} \int \frac{u^{m-1}}{\sqrt{au+b}} du$$

$$67) \int \frac{du}{u^m \sqrt{au+b}} = -\frac{\sqrt{au+b}}{(m-1)b u^{m-1}} - \frac{(2m-3)a}{(2m-2)b} \int \frac{du}{u^{m-1} \sqrt{au+b}}$$

$$68) \int u^m \sqrt{au+b} du = \frac{2u^m}{(2m+3)a} (au+b)^{3/2} - \frac{2mb}{(2m+3)a} \int u^{m-1} \sqrt{au+b} du$$

$$69) \int \frac{\sqrt{au+b}}{u^m} du = -\frac{\sqrt{au+b}}{(m-1)u^{m-1}} + \frac{a}{2(m-1)} \int \frac{du}{u^{m-1} \sqrt{au+b}}$$

$$70) \int \frac{\sqrt{au+b}}{u^m} du = \frac{-(au+b)^{3/2}}{(m-1)bu^{m-1}} - \frac{(2m-5)a}{(2m-2)b} \int \frac{\sqrt{au+b}}{u^{m-1}} du$$

$$71) \int (au+b)^{m/2} du = \frac{2(au+b)^{(m+2)/2}}{a(m+2)}$$

$$72) \int u(au+b)^{m/2} du = \frac{2(au+b)^{(m+4)/2}}{a^2(m+4)} - \frac{2b(au+b)^{(m+2)/2}}{a^2(m+2)}$$

$$73) \int u^2(au+b)^{m/2} du = \frac{2(au+b)^{(m+6)/2}}{a^3(m+6)} - \frac{4b(au+b)^{(m+4)/2}}{a^3(m+4)} + \frac{2b^2(au+b)^{(m+2)/2}}{a^3(m+2)}$$

$$74) \int \frac{(au+b)^{m/2}}{u} du = \frac{2(au+b)^{m/2}}{m} + b \int \frac{(au+b)^{(m-2)/2}}{u} du$$

$$75) \int \frac{(au+b)^{m/2}}{u^2} du = -\frac{(au+b)^{(m+2)/2}}{bu} + \frac{ma}{2b} \int \frac{(au+b)^{m/2}}{u} du$$

$$76) \int \frac{du}{u(au+b)^{m/2}} = \frac{2}{b(m-2)(au+b)^{(m-2)/2}} + \frac{1}{b} \int \frac{du}{u(au+b)^{(m-2)/2}}$$

Integrales con $u^2 + a^2$.

$$77) \int \frac{du}{u^2+a^2} = \frac{1}{a} \tan^{-1} \frac{u}{a}$$

$$78) \int \frac{u du}{u^2+a^2} = \frac{1}{2} \ln(u^2+a^2)$$

$$79) \int \frac{u^2 du}{u^2+a^2} = u - a \tan^{-1} \frac{u}{a}$$

$$80) \int \frac{u^3 du}{u^2+a^2} = \frac{u^2}{2} - \frac{a^2}{2} \ln(u^2+a^2)$$

$$81) \int \frac{du}{u(u^2+a^2)} = \frac{1}{2a^2} \ln \left(\frac{u^2}{u^2+a^2} \right)$$

$$82) \int \frac{du}{u^2(u^2+a^2)} = -\frac{1}{a^2 u} - \frac{1}{a^3} \tan^{-1} \frac{u}{a}$$

$$83) \int \frac{du}{u^3(u^2+a^2)} = -\frac{1}{2a^2 u^2} - \frac{1}{2a^4} \ln \left(\frac{u^2}{u^2+a^2} \right)$$

$$84) \int \frac{du}{(u^2+a^2)^2} = \frac{u}{2a^2(u^2+a^2)} + \frac{1}{2a^3} \tan^{-1} \frac{u}{a}$$

$$85) \int \frac{u du}{(u^2+a^2)^2} = \frac{-1}{2(u^2+a^2)}$$

$$86) \int \frac{u^2 du}{(u^2+a^2)^2} = \frac{-u}{2(u^2+a^2)} + \frac{1}{2a} \tan^{-1} \frac{u}{a}$$

$$87) \int \frac{u^3 du}{(u^2+a^2)^2} = \frac{a^2}{2(u^2+a^2)} + \frac{1}{2} \ln(u^2+a^2)$$

$$88) \int \frac{du}{u(u^2+a^2)^2} = \frac{1}{2a^2(u^2+a^2)} + \frac{1}{2a^4} \ln \left(\frac{u^2}{u^2+a^2} \right)$$

$$89) \int \frac{du}{u^2(u^2+a^2)^2} = -\frac{1}{a^4 u} - \frac{u}{2a^4(u^2+a^2)} - \frac{3}{2a^5} \tan^{-1} \frac{u}{a}$$

$$90) \int \frac{du}{u^3(u^2+a^2)^2} = -\frac{1}{2a^4 u^2} - \frac{1}{2a^4(u^2+a^2)} - \frac{1}{a^6} \ln \left(\frac{u^2}{u^2+a^2} \right)$$

$$91) \int \frac{du}{(u^2+a^2)^n} = \frac{u}{2a^2(n-1)(u^2+a^2)^{n-1}} + \frac{2n-3}{(2n-2)a^2} \int \frac{du}{(u^2+a^2)^{n-1}}$$

$$92) \int \frac{u du}{(u^2+a^2)^n} = \frac{-1}{2(n-1)(u^2+a^2)^{n-1}}$$

$$93) \int \frac{du}{u(u^2+a^2)^n} = \frac{1}{2a^2(n-1)(u^2+a^2)^{n-1}} + \frac{1}{a^2} \int \frac{du}{u(u^2+a^2)^{n-1}}$$

$$94) \int \frac{u^m du}{(u^2+a^2)^n} = \int \frac{u^{m-2} du}{(u^2+a^2)^{n-1}} - a^2 \int \frac{u^{m-2} du}{(u^2+a^2)^n}$$

$$95) \int \frac{du}{u^m(u^2+a^2)^n} = \frac{1}{a^2} \int \frac{du}{u^m(u^2+a^2)^{n-1}} - \frac{1}{a^2} \int \frac{du}{u^{m-2}(u^2+a^2)^n}$$

Integrales con $u^2 - a^2$.

$$96) \int \frac{du}{u^2 - a^2} = \frac{1}{2a} \ln \left(\frac{u-a}{u+a} \right) = -\frac{1}{a} \coth^{-1} \frac{u}{a}$$

$$97) \int \frac{u du}{u^2 - a^2} = \frac{1}{2} \ln (u^2 - a^2)$$

$$98) \int \frac{u^2 du}{u^2 - a^2} = u + \frac{a}{2} \ln \left(\frac{u-a}{u+a} \right)$$

$$99) \int \frac{u^3 du}{u^2 - a^2} = \frac{u^2}{2} + \frac{a^2}{2} \ln (u^2 - a^2)$$

$$100) \int \frac{du}{u(u^2 - a^2)} = \frac{1}{2a^2} \ln \left(\frac{u^2 - a^2}{u^2} \right)$$

$$101) \int \frac{du}{u^2(u^2 - a^2)} = \frac{1}{a^2 u} + \frac{1}{2a^3} \ln \left(\frac{u-a}{u+a} \right)$$

$$102) \int \frac{du}{u^3(u^2 - a^2)} = \frac{1}{2a^2 u^2} - \frac{1}{2a^4} \ln \left(\frac{u^2}{u^2 - a^2} \right)$$

$$103) \int \frac{du}{(u^2 - a^2)^2} = \frac{-u}{2a^2(u^2 - a^2)} - \frac{1}{4a^3} \ln \left(\frac{u-a}{u+a} \right)$$

$$104) \int \frac{u du}{(u^2 - a^2)^2} = \frac{-1}{2(u^2 - a^2)}$$

$$105) \int \frac{u^2 du}{(u^2 - a^2)^2} = \frac{-u}{2(u^2 - a^2)} + \frac{1}{4a} \ln \left(\frac{u-a}{u+a} \right)$$

$$106) \int \frac{u^3 du}{(u^2 - a^2)^2} = \frac{-a}{2(u^2 - a^2)} + \frac{1}{2} \ln (u^2 - a^2)$$

$$107) \int \frac{du}{u(u^2 - a^2)^2} = \frac{-1}{2a^2(u^2 - a^2)} + \frac{1}{2a^4} \ln \left(\frac{u^2}{u^2 - a^2} \right)$$

$$108) \int \frac{du}{u^2(u^2 - a^2)^2} = -\frac{1}{a^4 u} - \frac{u}{2a^4(u^2 - a^2)} - \frac{3}{4a^5} \ln \left(\frac{u-a}{u+a} \right)$$

$$109) \int \frac{du}{u^3(u^2 - a^2)^2} = -\frac{1}{2a^4 u^2} - \frac{1}{2a^4(u^2 - a^2)} + \frac{1}{a^6} \ln \left(\frac{u^2}{u^2 - a^2} \right)$$

$$110) \int \frac{du}{(u^2 - a^2)^n} = \frac{-u}{2a^2(n-1)(u^2 - a^2)^{n-1}} - \frac{2n-3}{(2n-2)a^2} \int \frac{du}{(u^2 - a^2)^{n-1}}$$

$$111) \int \frac{u du}{(u^2 - a^2)^n} = \frac{-1}{2(n-1)(u^2 - a^2)^{n-1}}$$

$$112) \int \frac{du}{u(u^2 - a^2)^n} = \frac{-1}{2a^2(n-1)(u^2 - a^2)^{n-1}} - \frac{1}{a^2} \int \frac{du}{u(u^2 - a^2)^{n-1}}$$

$$113) \int \frac{u^m du}{(u^2 - a^2)^n} = \int \frac{u^{m-2} du}{(u^2 - a^2)^{n-1}} + a^2 \int \frac{u^{m-2} du}{(u^2 - a^2)^n}$$

$$114) \int \frac{du}{u^m(u^2 - a^2)^n} = \frac{1}{a^2} \int \frac{du}{u^{m-2}(u^2 - a^2)^n} + \frac{1}{a^2} \int \frac{du}{u^m(u^2 - a^2)^{n-1}}$$

Integrales con $a^2 - u^2, u^2 < a^2$.

$$115) \int \frac{du}{a^2 - u^2} = \frac{1}{2a} \ln \left(\frac{a+u}{a-u} \right) = \frac{1}{a} \tanh^{-1} \frac{u}{a}$$

$$116) \int \frac{u du}{a^2 - u^2} = -\frac{1}{2} \ln (a^2 - u^2)$$

$$117) \int \frac{u^2 du}{a^2 - u^2} = -u + \frac{a}{2} \ln \left(\frac{a+u}{a-u} \right)$$

$$118) \int \frac{u^3 du}{a^2 - u^2} = -\frac{u^2}{2} - \frac{a^2}{2} \ln (a^2 - u^2)$$

$$119) \int \frac{du}{u(a^2 - u^2)} = \frac{1}{2a^2} \ln \left(\frac{u^2}{a^2 - u^2} \right)$$

$$120) \int \frac{du}{u^2(a^2 - u^2)} = \frac{1}{a^2 u} + \frac{1}{2a^3} \ln \left(\frac{a+u}{a-u} \right)$$

$$121) \int \frac{du}{u^3(a^2 - u^2)} = -\frac{1}{2a^2 u^2} + \frac{1}{2a^4} \ln \left(\frac{u^2}{a^2 - u^2} \right)$$

$$122) \int \frac{du}{(a^2 - u^2)^2} = \frac{u}{2a^2(a^2 - u^2)} + \frac{1}{4a^3} \ln \left(\frac{a+u}{a-u} \right)$$

$$123) \int \frac{u du}{(a^2 - u^2)^2} = \frac{1}{2(a^2 - u^2)}$$

$$124) \int \frac{u^2 du}{(a^2 - u^2)^2} = \frac{u}{2(a^2 - u^2)} - \frac{1}{4a} \ln \left(\frac{a+u}{a-u} \right)$$

$$125) \int \frac{u^3 du}{(a^2 - u^2)^2} = \frac{a^2}{2(a^2 - u^2)} + \frac{1}{2} \ln (a^2 - u^2)$$

$$126) \int \frac{du}{u(a^2 - u^2)^2} = \frac{1}{2a^2(a^2 - u^2)} + \frac{1}{2a^4} \ln \left(\frac{u^2}{a^2 - u^2} \right)$$

$$127) \int \frac{du}{u^2(a^2 - u^2)^2} = -\frac{1}{a^4 u} + \frac{u}{2a^4(a^2 - u^2)} + \frac{3}{4a^5} \ln \left(\frac{a+u}{a-u} \right)$$

$$128) \int \frac{du}{u^3(a^2 - u^2)^2} = -\frac{1}{2a^4 u^2} + \frac{1}{2a^4(a^2 - u^2)} + \frac{1}{a^6} \ln \left(\frac{u^2}{a^2 - u^2} \right)$$

$$129) \int \frac{dx}{(a^2 - x^2)^n} = \frac{x}{2(n-1)a^2(a^2 - x^2)^{n-1}} + \frac{2n-3}{(2n-2)a^2} \int \frac{dx}{(a^2 - x^2)^{n-1}}$$

$$130) \int \frac{x dx}{(a^2 - x^2)^n} = \frac{1}{2(n-1)(a^2 - x^2)^{n-1}}$$

Integrales con $\sqrt{u^2 + a^2}$.

$$131) \int \sqrt{u^2 + a^2} du = \frac{u\sqrt{u^2 + a^2}}{2} + \frac{a^2}{2} \ln (u + \sqrt{u^2 + a^2})$$

$$132) \int u \sqrt{u^2 + a^2} du = \frac{(u^2 + a^2)^{3/2}}{3}$$

$$133) \int u^2 \sqrt{u^2 + a^2} du = \frac{u(u^2 + a^2)^{3/2}}{4} - \frac{a^2 u \sqrt{u^2 + a^2}}{8} - \frac{a^4}{8} \ln (u + \sqrt{u^2 + a^2})$$

$$134) \int u^3 \sqrt{u^2 + a^2} du = \frac{(u^2 + a^2)^{5/2}}{5} - \frac{a^2 (u^2 + a^2)^{3/2}}{3}$$

$$135) \int \frac{du}{\sqrt{u^2 + a^2}} = \ln (u + \sqrt{u^2 + a^2}) = \sinh^{-1} \frac{u}{a}$$

$$136) \int \frac{u du}{\sqrt{u^2 + a^2}} = \sqrt{u^2 + a^2}$$

$$137) \int \frac{u^2 du}{\sqrt{u^2 + a^2}} = \frac{u\sqrt{u^2 + a^2}}{2} - \frac{a^2}{2} \ln (u + \sqrt{u^2 + a^2})$$

$$138) \int \frac{u^3 du}{\sqrt{u^2 + a^2}} = \frac{(u^2 + a^2)^{3/2}}{3} - a^2 \sqrt{u^2 + a^2}$$

$$139) \int \frac{du}{u \sqrt{u^2 + a^2}} = -\frac{1}{a} \ln \left(\frac{a + \sqrt{u^2 + a^2}}{u} \right)$$

$$140) \int \frac{du}{u^2 \sqrt{u^2 + a^2}} = -\frac{\sqrt{u^2 + a^2}}{a^2 u}$$

$$141) \int \frac{du}{u^3 \sqrt{u^2 + a^2}} = -\frac{\sqrt{u^2 + a^2}}{2a^2 u^2} + \frac{1}{2a^3} \ln \left(\frac{a + \sqrt{u^2 + a^2}}{u} \right)$$

$$142) \int \frac{\sqrt{u^2 + a^2}}{u} du = \sqrt{u^2 + a^2} - a \ln \left(\frac{a + \sqrt{u^2 + a^2}}{u} \right)$$

$$143) \int \frac{\sqrt{u^2 + a^2}}{u^2} du = -\frac{\sqrt{u^2 + a^2}}{u} + \ln (u + \sqrt{u^2 + a^2})$$

$$144) \int \frac{\sqrt{u^2 + a^2}}{u^3} du = -\frac{\sqrt{u^2 + a^2}}{2u^2} - \frac{1}{2a} \ln \left(\frac{a + \sqrt{u^2 + a^2}}{u} \right)$$

$$145) \int \frac{du}{(u^2 + a^2)^{3/2}} = \frac{u}{a^2 \sqrt{u^2 + a^2}}$$

$$146) \int \frac{u du}{(u^2 + a^2)^{3/2}} = \frac{-1}{\sqrt{u^2 + a^2}}$$

$$147) \int \frac{u^2 du}{(u^2 + a^2)^{3/2}} = \frac{-u}{\sqrt{u^2 + a^2}} + \ln (u + \sqrt{u^2 + a^2})$$

$$148) \int \frac{u^3 du}{(u^2 + a^2)^{3/2}} = \sqrt{u^2 + a^2} + \frac{a^2}{\sqrt{u^2 + a^2}}$$

$$149) \int \frac{du}{u(u^2 + a^2)^{3/2}} = \frac{1}{a^2 \sqrt{u^2 + a^2}} - \frac{1}{a^3} \ln \left(\frac{a + \sqrt{u^2 + a^2}}{u} \right)$$

$$150) \int \frac{du}{u^2(u^2+a^2)^{3/2}} = -\frac{\sqrt{u^2+a^2}}{a^4 u} - \frac{u}{a^4 \sqrt{u^2+a^2}}$$

$$151) \int \frac{du}{u^3(u^2+a^2)^{3/2}} = \frac{-1}{2a^2 u^2 \sqrt{u^2+a^2}} - \frac{3}{2a^4 \sqrt{u^2+a^2}} + \frac{3}{2a^5} \ln \left(\frac{a+\sqrt{u^2+a^2}}{u} \right)$$

$$152) \int (u^2+a^2)^{3/2} du = \frac{u(u^2+a^2)^{3/2}}{4} + \frac{3a^2 u \sqrt{u^2+a^2}}{8} + \frac{3}{8} a^4 \ln(u + \sqrt{u^2+a^2})$$

$$153) \int u(u^2+a^2)^{3/2} du = \frac{(u^2+a^2)^{5/2}}{5}$$

$$154) \int u^2(u^2+a^2)^{3/2} du = \frac{u(u^2+a^2)^{5/2}}{6} - \frac{a^2 u(u^2+a^2)^{3/2}}{24} - \frac{a^4 u \sqrt{u^2+a^2}}{16} - \frac{a^6}{16} \ln(u + \sqrt{u^2+a^2})$$

$$155) \int \frac{(u^2+a^2)^{3/2}}{u} du = \frac{(u^2+a^2)^{3/2}}{3} + a^2 \sqrt{u^2+a^2} - a^3 \ln \left(\frac{a+\sqrt{u^2+a^2}}{u} \right)$$

$$156) \int \frac{(u^2+a^2)^{3/2}}{u^2} du = -\frac{(u^2+a^2)^{3/2}}{u} + \frac{3u\sqrt{u^2+a^2}}{2} + \frac{3}{2} a^2 \ln(u + \sqrt{u^2+a^2})$$

$$157) \int \frac{(u^2+a^2)^{3/2}}{u^3} du = -\frac{(u^2+a^2)^{3/2}}{2u^2} + \frac{3}{2} \sqrt{u^2+a^2} - \frac{3}{2} a \ln \left(\frac{a+\sqrt{u^2+a^2}}{u} \right)$$

Integrales con $\sqrt{u^2-a^2}$.

$$158) \int \frac{du}{\sqrt{u^2-a^2}} = \ln(u + \sqrt{u^2-a^2})$$

$$159) \int \frac{u du}{\sqrt{u^2-a^2}} = \sqrt{u^2-a^2}$$

$$160) \int \frac{u^2 du}{\sqrt{u^2-a^2}} = \frac{u\sqrt{u^2-a^2}}{2} + \frac{a^2}{2} \ln(u + \sqrt{u^2-a^2})$$

$$161) \int \frac{u^3 du}{\sqrt{u^2-a^2}} = \frac{(u^2-a^2)^{3/2}}{3} + a^2 \sqrt{u^2-a^2}$$

$$162) \int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a} \sec^{-1} \left(\frac{u}{a} \right)$$

$$163) \int \frac{du}{u^2 \sqrt{u^2-a^2}} = \frac{\sqrt{u^2-a^2}}{a^2 u}$$

$$164) \int \frac{du}{u^3 \sqrt{u^2-a^2}} = \frac{\sqrt{u^2-a^2}}{2a^2 u^2} + \frac{1}{2a^3} \sec^{-1} \left(\frac{u}{a} \right)$$

$$165) \int \sqrt{u^2-a^2} du = \frac{u\sqrt{u^2-a^2}}{2} - \frac{a^2}{2} \ln(u + \sqrt{u^2-a^2})$$

$$166) \int u\sqrt{u^2-a^2} du = \frac{(u^2-a^2)^{3/2}}{3}$$

$$167) \int u^2 \sqrt{u^2-a^2} du = \frac{u(u^2-a^2)^{3/2}}{4} + \frac{a^2 u \sqrt{u^2-a^2}}{8} - \frac{a^4}{8} \ln(u + \sqrt{u^2-a^2})$$

$$168) \int u^3 \sqrt{u^2-a^2} du = \frac{(u^2-a^2)^{5/2}}{5} + \frac{a^2 (u^2-a^2)^{3/2}}{3}$$

$$169) \int \frac{\sqrt{u^2-a^2}}{u} du = \sqrt{u^2-a^2} - a \sec^{-1} \frac{u}{a}$$

$$170) \int \frac{\sqrt{u^2-a^2}}{u^2} du = -\frac{\sqrt{u^2-a^2}}{u} + \ln(u + \sqrt{u^2-a^2})$$

$$171) \int \frac{\sqrt{u^2-a^2}}{u^3} du = -\frac{\sqrt{u^2-a^2}}{2u^2} + \frac{1}{2a} \sec^{-1} \frac{u}{a}$$

$$172) \int \frac{du}{(u^2-a^2)^{3/2}} = -\frac{u}{a^2 \sqrt{u^2-a^2}}$$

$$173) \int \frac{u du}{(u^2-a^2)^{3/2}} = \frac{-1}{\sqrt{u^2-a^2}}$$

$$174) \int \frac{u^2 du}{(u^2-a^2)^{3/2}} = -\frac{u}{\sqrt{u^2-a^2}} + \ln(u + \sqrt{u^2-a^2})$$

$$175) \int \frac{u^3 du}{(u^2-a^2)^{3/2}} = \sqrt{u^2-a^2} - \frac{a^2}{\sqrt{u^2-a^2}}$$

$$176) \int \frac{du}{u(u^2-a^2)^{3/2}} = \frac{-1}{a^2 \sqrt{u^2-a^2}} - \frac{1}{a^3} \sec^{-1} \frac{u}{a}$$

$$177) \int \frac{du}{u^2(u^2-a^2)^{3/2}} = -\frac{\sqrt{u^2-a^2}}{a^4 u} - \frac{u}{a^4 \sqrt{u^2-a^2}}$$

$$178) \int \frac{du}{u^3(u^2-a^2)^{3/2}} = \frac{1}{2a^2 u^2 \sqrt{u^2-a^2}} - \frac{3}{2a^4 \sqrt{u^2-a^2}} - \frac{3}{2a^5} \sec^{-1} \frac{u}{a}$$

$$179) \int (u^2-a^2)^{3/2} du = \frac{u(u^2-a^2)^{3/2}}{4} - \frac{3a^2 u \sqrt{u^2-a^2}}{8} + \frac{3}{8} a^4 \ln(u + \sqrt{u^2-a^2})$$

$$180) \int u(u^2-a^2)^{3/2} du = \frac{(u^2-a^2)^{5/2}}{5}$$

$$181) \int u^2(u^2-a^2)^{3/2} du = \frac{u(u^2-a^2)^{5/2}}{6} + \frac{a^2 u(u^2-a^2)^{3/2}}{24} - \frac{a^4 u \sqrt{u^2-a^2}}{16} + \frac{a^6}{16} \ln(u + \sqrt{u^2-a^2})$$

$$182) \int u^3(u^2-a^2)^{3/2} du = \frac{(u^2-a^2)^{7/2}}{7} + \frac{a^2 (u^2-a^2)^{5/2}}{5}$$

$$183) \int \frac{(u^2-a^2)^{3/2}}{u} du = \frac{(u^2-a^2)^{3/2}}{3} - a^2 \sqrt{u^2-a^2} + a^3 \sec^{-1} \frac{u}{a}$$

$$184) \int \frac{(u^2-a^2)^{3/2}}{u^2} du = -\frac{(u^2-a^2)^{3/2}}{u} + \frac{3u\sqrt{u^2-a^2}}{2} - \frac{3}{2} a^2 \ln(u + \sqrt{u^2-a^2})$$

$$185) \int \frac{(u^2-a^2)^{3/2}}{u^3} du = -\frac{(u^2-a^2)^{3/2}}{2u^2} + \frac{3\sqrt{u^2-a^2}}{2} - \frac{3}{2} a \sec^{-1} \frac{u}{a}$$

Integrales con $\sqrt{a^2-u^2}$.

$$186) \int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1} \frac{u}{a}$$

$$187) \int \frac{u du}{\sqrt{a^2-u^2}} = -\sqrt{a^2-u^2}$$

$$188) \int \frac{u^2 du}{\sqrt{a^2-u^2}} = -\frac{u\sqrt{a^2-u^2}}{2} + \frac{a^2}{2} \sin^{-1} \frac{u}{a}$$

$$189) \int \frac{u^3 du}{\sqrt{a^2-u^2}} = \frac{(a^2-u^2)^{3/2}}{3} - a^2 \sqrt{a^2-u^2}$$

$$190) \int \frac{du}{u\sqrt{a^2-u^2}} = -\frac{1}{a} \ln \left(\frac{a+\sqrt{a^2-u^2}}{u} \right)$$

$$191) \int \frac{du}{u^2 \sqrt{a^2-u^2}} = -\frac{\sqrt{a^2-u^2}}{a^2 u}$$

$$192) \int \frac{du}{u^3 \sqrt{a^2-u^2}} = -\frac{\sqrt{a^2-u^2}}{2a^2 u^2} - \frac{1}{2a^3} \ln \left(\frac{a+\sqrt{a^2-u^2}}{u} \right)$$

$$193) \int \sqrt{a^2-u^2} du = \frac{u\sqrt{a^2-u^2}}{2} + \frac{a^2}{2} \sin^{-1} \frac{u}{a}$$

$$194) \int u\sqrt{a^2-u^2} du = -\frac{(a^2-u^2)^{3/2}}{3}$$

$$195) \int u^2 \sqrt{a^2-u^2} du = -\frac{u(a^2-u^2)^{3/2}}{4} + \frac{a^2 u \sqrt{a^2-u^2}}{8} + \frac{a^4}{8} \sin^{-1} \frac{u}{a}$$

$$196) \int u^3 \sqrt{a^2-u^2} du = \frac{(a^2-u^2)^{5/2}}{5} - \frac{a^2 (a^2-u^2)^{3/2}}{3}$$

$$197) \int \frac{\sqrt{a^2-u^2}}{u} du = \sqrt{a^2-u^2} - a \ln \left(\frac{a+\sqrt{a^2-u^2}}{u} \right)$$

$$198) \int \frac{\sqrt{a^2-u^2}}{u^2} du = -\frac{\sqrt{a^2-u^2}}{u} - \operatorname{sen}^{-1} \frac{u}{a}$$

$$199) \int \frac{\sqrt{a^2-u^2}}{u^3} du = -\frac{\sqrt{a^2-u^2}}{2u^2} + \frac{1}{2a} \ln \left(\frac{a+\sqrt{a^2-u^2}}{u} \right)$$

$$200) \int \frac{du}{(a^2-u^2)^{3/2}} = \frac{u}{a^2\sqrt{a^2-u^2}}$$

$$201) \int \frac{udu}{(a^2-u^2)^{3/2}} = \frac{1}{\sqrt{a^2-u^2}}$$

$$202) \int \frac{u^2 du}{(a^2-u^2)^{3/2}} = \frac{u}{\sqrt{a^2-u^2}} - \operatorname{sen}^{-1} \frac{u}{a}$$

$$203) \int \frac{u^3 du}{(a^2-u^2)^{3/2}} = \sqrt{a^2-u^2} + \frac{a^2}{\sqrt{a^2-u^2}}$$

$$204) \int \frac{du}{u(a^2-u^2)^{3/2}} = \frac{1}{a^2\sqrt{a^2-u^2}} - \frac{1}{a^3} \ln \left(\frac{a+\sqrt{a^2-u^2}}{u} \right)$$

$$205) \int \frac{du}{u^2(a^2-u^2)^{3/2}} = -\frac{\sqrt{a^2-u^2}}{a^4 u} + \frac{u}{a^4\sqrt{a^2-u^2}}$$

$$206) \int \frac{du}{u^3(a^2-u^2)^{3/2}} = \frac{-1}{2a^2 u^2 \sqrt{a^2-u^2}} + \frac{3}{2a^4 \sqrt{a^2-u^2}} - \frac{3}{2a^5} \ln \left(\frac{a+\sqrt{a^2-u^2}}{u} \right)$$

$$207) \int (a^2-u^2)^{3/2} du = \frac{u(a^2-u^2)^{3/2}}{4} + \frac{3a^2 u \sqrt{a^2-u^2}}{8} + \frac{3}{8} a^4 \operatorname{sen}^{-1} \frac{u}{a}$$

$$208) \int u (a^2-u^2)^{3/2} du = -\frac{(a^2-u^2)^{5/2}}{5}$$

$$209) \int u^2 (a^2-u^2)^{3/2} du = -\frac{u(a^2-u^2)^{5/2}}{6} + \frac{a^2 u (a^2-u^2)^{3/2}}{24} + \frac{a^4 u \sqrt{a^2-u^2}}{16} + \frac{a^6}{16} \operatorname{sen}^{-1} \frac{u}{a}$$

$$210) \int u^3 (a^2-u^2)^{3/2} du = \frac{(a^2-u^2)^{7/2}}{7} - \frac{a^2 (a^2-u^2)^{5/2}}{5}$$

$$211) \int \frac{(a^2-u^2)^{3/2}}{u} du = \frac{(a^2-u^2)^{3/2}}{3} + a^2 \sqrt{a^2-u^2} - a^3 \ln \left(\frac{a+\sqrt{a^2-u^2}}{u} \right)$$

$$212) \int \frac{(a^2-u^2)^{3/2}}{u^2} du = -\frac{(a^2-u^2)^{3/2}}{u} - \frac{3u\sqrt{a^2-u^2}}{2} + \frac{3}{2} a^2 \operatorname{sen}^{-1} \frac{u}{a}$$

$$213) \int \frac{(a^2-u^2)^{3/2}}{u^3} du = -\frac{(a^2-u^2)^{3/2}}{2u^2} - \frac{3\sqrt{a^2-u^2}}{2} + \frac{3}{2} a \ln \left(\frac{a+\sqrt{a^2-u^2}}{u} \right)$$

Integrales con $au^2 + bu + c$.

$$214) \int \frac{du}{au^2+bu+c} = \begin{cases} \frac{2}{\sqrt{4ac-b^2}} \tan^{-1} \left(\frac{2au+b}{\sqrt{4ac-b^2}} \right) \\ \frac{1}{\sqrt{b^2-4ac}} \ln \left(\frac{2au+b-\sqrt{b^2-4ac}}{2au+b+\sqrt{b^2-4ac}} \right) \end{cases}$$

$$215) \int \frac{udu}{au^2+bu+c} = \frac{1}{2a} \ln (au^2+bu+c) - \frac{b}{2a} \int \frac{du}{au^2+bu+c}$$

$$216) \int \frac{u^2 du}{au^2+bu+c} = \frac{u}{a} - \frac{b}{2a^2} \ln (au^2+bu+c) + \frac{b^2-2ac}{2a^2} \int \frac{du}{au^2+bu+c}$$

$$217) \int \frac{du}{u(au^2+bu+c)} = \frac{1}{2c} \ln \left(\frac{u^2}{au^2+bu+c} \right) - \frac{b}{2c} \int \frac{du}{au^2+bu+c}$$

$$218) \int \frac{du}{u^2(au^2+bu+c)} = \frac{b}{2c^2} \ln \left(\frac{au^2+bu+c}{u^2} \right) - \frac{1}{cu}$$

$$+ \frac{b^2-2ac}{2c^2} \int \frac{du}{au^2+bu+c}$$

$$219) \int \frac{du}{(au^2+bu+c)^2} = \frac{2au+b}{(4ac-b^2)(au^2+bu+c)} + \frac{2a}{4ac-b^2} \int \frac{du}{au^2+bu+c}$$

$$220) \int \frac{udu}{(au^2+bu+c)^2} = -\frac{bu+2c}{(4ac-b^2)(au^2+bu+c)} - \frac{b}{4ac-b^2} \int \frac{du}{au^2+bu+c}$$

$$221) \int \frac{u^2 du}{(au^2+bu+c)^2} = \frac{(b^2-2ac)u+bc}{a(4ac-b^2)(au^2+bu+c)} + \frac{2c}{4ac-b^2} \int \frac{du}{au^2+bu+c}$$

$$222) \int \frac{du}{u(au^2+bu+c)^2} = \frac{1}{2c(au^2+bu+c)} - \frac{b}{2c} \int \frac{du}{(au^2+bu+c)^2} + \frac{1}{c} \int \frac{du}{u(au^2+bu+c)}$$

$$223) \int \frac{du}{u^2(au^2+bu+c)^2} = -\frac{1}{cu(au^2+bu+c)} - \frac{3a}{c} \int \frac{du}{(au^2+bu+c)^2} - \frac{2b}{c} \int \frac{du}{u(au^2+bu+c)^2}$$

$$224) \int \frac{u^m du}{au^2+bu+c} = \frac{u^{m-1}}{(m-1)a} - \frac{c}{a} \int \frac{u^{m-2} du}{au^2+bu+c} - \frac{b}{a} \int \frac{u^{m-1} du}{au^2+bu+c}$$

$$225) \int \frac{du}{u^n(au^2+bu+c)} = -\frac{1}{(n-1)cu^{n-1}} - \frac{b}{c} \int \frac{du}{u^{n-1}(au^2+bu+c)} - \frac{a}{c} \int \frac{du}{u^{n-2}(au^2+bu+c)}$$

Integrales con $u^3 + a^3$.

$$226) \int \frac{du}{u^3+a^3} = \frac{1}{6a^2} \ln \frac{(u+a)^2}{u^2-au+a^2} + \frac{1}{a\sqrt{3}} \tan^{-1} \frac{2u-a}{a\sqrt{3}}$$

$$227) \int \frac{udu}{u^3+a^3} = \frac{1}{6a} \ln \frac{u^2-au+a^2}{(u+a)^2} + \frac{1}{a\sqrt{3}} \tan^{-1} \frac{2u-a}{a\sqrt{3}}$$

$$228) \int \frac{u^2 du}{u^3+a^3} = \frac{1}{3} \ln (u^3+a^3)$$

$$229) \int \frac{du}{u(u^3+a^3)} = \frac{1}{3a^3} \ln \left(\frac{u^3}{u^3+a^3} \right)$$

$$230) \int \frac{du}{u^2(u^3+a^3)} = -\frac{1}{a^3 u} - \frac{1}{6a^4} \ln \frac{u^2-au+a^2}{(u+a)^2} - \frac{1}{a^4\sqrt{3}} \tan^{-1} \frac{2u-a}{a\sqrt{3}}$$

$$231) \int \frac{du}{(u^3+a^3)^2} = \frac{u}{3a^3(u^3+a^3)} + \frac{1}{9a^5} \ln \frac{(u+a)^2}{u^2-au+a^2} + \frac{2}{3a^5\sqrt{3}} \tan^{-1} \frac{2u-a}{a\sqrt{3}}$$

$$232) \int \frac{udu}{(u^3+a^3)^2} = \frac{u^2}{3a^3(u^3+a^3)} + \frac{1}{18a^4} \ln \frac{u^2-au+a^2}{(u+a)^2} + \frac{1}{3a^4\sqrt{3}} \tan^{-1} \frac{2u-a}{a\sqrt{3}}$$

$$233) \int \frac{u^2 du}{(u^3+a^3)^2} = -\frac{1}{3(u^3+a^3)}$$

$$234) \int \frac{du}{u(u^3+a^3)^2} = \frac{1}{3a^3(u^3+a^3)} + \frac{1}{3a^6} \ln \left(\frac{u^3}{u^3+a^3} \right)$$

$$235) \int \frac{du}{u^2(u^3+a^3)^2} = -\frac{1}{a^6 u} - \frac{u^2}{3a^6(u^3+a^3)} - \frac{4}{3a^6} \int \frac{udu}{u^3+a^3}$$

$$236) \int \frac{u^m du}{u^3+a^3} = \frac{u^{m-2}}{m-2} - a^3 \int \frac{u^{m-3} du}{u^3+a^3}$$

$$237) \int \frac{du}{u^n(u^3+a^3)} = \frac{-1}{a^3(n-1)u^{n-1}} - \frac{1}{a^3} \int \frac{du}{u^{n-3}(u^3+a^3)}$$

Integrales con $u^3 \pm a^3$.

$$238) \int \frac{du}{u^4+a^4} = \frac{1}{4a^3\sqrt{2}} \ln \left(\frac{u^2+au\sqrt{2}+a^2}{u^2-au\sqrt{2}+a^2} \right) - \frac{1}{2a^3\sqrt{2}} \left[\tan^{-1} \left(1 - \frac{u\sqrt{2}}{a} \right) - \tan^{-1} \left(1 + \frac{u\sqrt{2}}{a} \right) \right]$$

$$239) \int \frac{u^2 du}{u^4+a^4} = \frac{1}{4a\sqrt{2}} \ln \left(\frac{u^2+au\sqrt{2}+a^2}{u^2-au\sqrt{2}+a^2} \right) - \frac{1}{2a\sqrt{2}} \left[\tan^{-1} \left(1 - \frac{u\sqrt{2}}{a} \right) - \tan^{-1} \left(1 + \frac{u\sqrt{2}}{a} \right) \right]$$

$$240) \int \frac{du}{u^2(u^4+a^4)} = -\frac{1}{a^4 u} - \frac{1}{4a^5\sqrt{2}} \ln \left(\frac{u^2+au\sqrt{2}+a^2}{u^2-au\sqrt{2}+a^2} \right) + \frac{1}{2a^5\sqrt{2}} \left[\tan^{-1} \left(1 - \frac{u\sqrt{2}}{a} \right) - \tan^{-1} \left(1 + \frac{u\sqrt{2}}{a} \right) \right]$$

$$241) \int \frac{u^3 du}{u^4+a^4} = \frac{1}{4} \ln(u^4+a^4)$$

$$242) \int \frac{du}{u(u^4+a^4)} = \frac{1}{4a^4} \ln \left(\frac{u^4}{u^4+a^4} \right)$$

$$243) \int \frac{u du}{u^4+a^4} = \frac{1}{2a^2} \tan^{-1} \frac{u^2}{a^2}$$

$$244) \int \frac{du}{u^3(u^4+a^4)} = -\frac{1}{2a^4 u^2} - \frac{1}{2a^6} \tan^{-1} \frac{u^2}{a^2}$$

$$245) \int \frac{du}{u^4-a^4} = \frac{1}{4a^3} \ln \left(\frac{u-a}{u+a} \right) - \frac{1}{2a^3} \tan^{-1} \frac{u}{a}$$

$$246) \int \frac{u du}{u^4-a^4} = \frac{1}{4a^2} \ln \left(\frac{u^2-a^2}{u^2+a^2} \right)$$

$$247) \int \frac{u^2 du}{u^4-a^4} = \frac{1}{4a} \ln \left(\frac{u-a}{u+a} \right) + \frac{1}{2a} \tan^{-1} \frac{u}{a}$$

$$248) \int \frac{u^3 du}{u^4-a^4} = \frac{1}{4} \ln(u^4-a^4)$$

$$249) \int \frac{du}{u(u^4-a^4)} = \frac{1}{4a^4} \ln \left(\frac{u^4-a^4}{u^4} \right)$$

$$250) \int \frac{du}{u^2(u^4-a^4)} = \frac{1}{a^4 u} + \frac{1}{4a^6} \ln \left(\frac{u-a}{u+a} \right) + \frac{1}{2a^6} \tan^{-1} \frac{u}{a}$$

$$251) \int \frac{du}{u^3(u^4-a^4)} = \frac{1}{2a^4 u^2} + \frac{1}{4a^6} \ln \left(\frac{u^2-a^2}{u^2+a^2} \right)$$

Integrales con $\sin(au)$.

$$252) \int \sin(au) du = -\frac{\cos(au)}{a}$$

$$253) \int u \sin(au) du = \frac{\sin(au)}{a^2} - \frac{u \cos(au)}{a}$$

$$254) \int u^2 \sin(au) du = \frac{2u}{a^2} \sin(au) + \left(\frac{2}{a^3} - \frac{u^2}{a} \right) \cos(au)$$

$$255) \int u^3 \sin(au) du = \left(\frac{3u^2}{a^2} - \frac{6}{a^4} \right) \sin(au) + \left(\frac{6u}{a^3} - \frac{u^3}{a} \right) \cos(au)$$

$$256) \int u^n \sin(au) du = -\frac{u^n \cos(au)}{a} + \frac{n}{a} \int u^{n-1} \cos(au) du$$

$$257) \int u^n \sin(au) du = -\frac{u^n \cos(au)}{a} + \frac{nu^{n-1}}{a^2} \sin(au) - \frac{n(n-1)}{a^2} \int u^{n-2} \sin(au) du$$

$$258) \int \sin^2(au) du = \frac{u}{2} - \frac{\sin(2au)}{4a}$$

$$259) \int \sin^3(au) du = -\frac{\cos(ax)}{a} + \frac{\cos^3(au)}{3a}$$

$$260) \int \sin^4(au) du = \frac{3u}{8} - \frac{\sin(2au)}{4a} + \frac{\sin(4au)}{32a}$$

$$261) \int u \sin^2(au) du = \frac{u^2}{4} - \frac{u \sin(2au)}{4a} - \frac{\cos(2au)}{8a^2}$$

$$262) \int \frac{\sin(au)}{u} du = au - \frac{(au)^3}{3 \cdot 3!} + \frac{(au)^5}{5 \cdot 5!} - \dots$$

$$263) \int \frac{\sin(au)}{u^2} du = -\frac{\sin(au)}{u} + a \int \frac{\cos(au)}{u} du$$

$$264) \int \frac{du}{\sin(au)} = \frac{1}{a} \ln [\csc(au) - \cot(au)] = \frac{1}{a} \ln \left[\tan \left(\frac{au}{2} \right) \right]$$

$$265) \int \frac{u du}{\sin(au)} = \frac{1}{a^2} \left\{ au + \frac{(au)^3}{18} + \frac{7(au)^5}{1800} + \dots + \frac{2(2^{2n-1}-1)B_n(au)^{2n+1}}{(2n+1)!} + \dots \right\}$$

$$266) \int \frac{du}{\sin^2(au)} = -\frac{1}{a} \cot(au)$$

$$267) \int \frac{du}{\sin^3(au)} = -\frac{\cos(au)}{2a \sin^2(au)} + \frac{1}{2a} \ln \left[\tan \left(\frac{au}{2} \right) \right]$$

$$268) \int \sin(pu) \sin(qu) du = \frac{\sin[(p-q)u]}{2(p-q)} - \frac{\sin[(p+q)u]}{2(p+q)}$$

$$269) \int \frac{du}{1-\sin(au)} = \frac{1}{a} \tan \left(\frac{\pi}{4} + \frac{au}{2} \right)$$

$$270) \int \frac{u du}{1-\sin(au)} = \frac{u}{a} \tan \left(\frac{\pi}{4} + \frac{au}{2} \right) + \frac{2}{a^2} \ln \sin \left(\frac{\pi}{4} - \frac{au}{2} \right)$$

$$271) \int \frac{du}{1+\sin(au)} = -\frac{1}{a} \tan \left(\frac{\pi}{4} - \frac{au}{2} \right)$$

$$272) \int \frac{u du}{1+\sin(au)} = -\frac{u}{a} \tan \left(\frac{\pi}{4} - \frac{au}{2} \right) + \frac{2}{a^2} \ln \sin \left(\frac{\pi}{4} + \frac{au}{2} \right)$$

$$273) \int \frac{du}{(1-\sin(au))^2} = \frac{1}{2a} \tan \left(\frac{\pi}{4} + \frac{au}{2} \right) + \frac{1}{6a} \tan^3 \left(\frac{\pi}{4} + \frac{au}{2} \right)$$

$$274) \int \frac{dx}{(1+\sin ax)^2} = -\frac{1}{2a} \tan \left(\frac{\pi}{4} - \frac{ax}{2} \right) - \frac{1}{6a} \tan^3 \left(\frac{\pi}{4} - \frac{ax}{2} \right)$$

Integrales con $\cos(au)$.

$$275) \int \cos(au) du = \frac{\sin(au)}{a}$$

$$276) \int u \cos(au) du = \frac{\cos(au)}{a^2} + \frac{u \sin(au)}{a}$$

$$277) \int u^2 \cos(au) du = \frac{2u}{a^2} \cos(au) + \left(\frac{u^2}{a} - \frac{2}{a^3} \right) \sin(au)$$

$$278) \int u^3 \cos(au) du = \left(\frac{3u^2}{a^2} - \frac{6}{a^4} \right) \cos(au) + \left(\frac{u^3}{a} - \frac{6u}{a^3} \right) \sin(au)$$

$$279) \int u^n \cos(au) du = \frac{u^n \sin(au)}{a} - \frac{n}{a} \int u^{n-1} \sin(au) du$$

$$280) \int u^n \cos(au) du = -\frac{u^n \sin(au)}{a} + \frac{nu^{n-1}}{a^2} \cos(au) - \frac{n(n-1)}{a^2} \int u^{n-2} \cos(au) du$$

$$281) \int \cos^2(au) du = \frac{u}{2} + \frac{\sin(2au)}{4a}$$

$$282) \int \cos^3(au) du = \frac{\sin(au)}{a} - \frac{\sin^3(au)}{3a}$$

$$283) \int \cos^4(au) du = \frac{3u}{8} + \frac{\sin 2(au)}{4a} + \frac{\sin 4(au)}{32a}$$

$$284) \int u \cos^2(au) du = \frac{u^2}{4} + \frac{u \sin 2(au)}{4a} + \frac{\cos 2(au)}{8a^2}$$

$$285) \int \frac{\cos(au)}{u} du = \ln u - \frac{(au)^2}{2 \cdot 2!} + \frac{(au)^4}{4 \cdot 4!} - \frac{(au)^6}{6 \cdot 6!} + \dots$$

$$286) \int \frac{\cos(au)}{u^2} du = -\frac{\cos(au)}{u} - a \int \frac{\sin(au)}{u} du$$

$$287) \int \frac{du}{\cos(au)} = \frac{1}{a} \ln [\sec(au) + \tan(au)] = \frac{1}{a} \ln \left[\tan \left(\frac{\pi}{4} + \frac{au}{2} \right) \right]$$

$$288) \int \frac{u du}{\cos(au)} = \frac{1}{a^2} \left\{ \frac{(au)^2}{2} + \frac{(au)^4}{8} + \frac{5(au)^6}{144} + \dots + \frac{E_n(au)^{2n+2}}{(2n+2)(2n)!} + \dots \right\}$$

$$289) \int \frac{du}{\cos^2(au)} = \frac{\tan(au)}{a}$$

$$290) \int \frac{du}{\cos^3(au)} = \frac{\sec(au)}{2a \cos^2(au)} + \frac{1}{2a} \ln \left[\tan \left(\frac{\pi}{4} + \frac{au}{2} \right) \right]$$

$$291) \int \cos(au) \cos(pu) du = \frac{\sec[(a-p)u]}{2(a-p)} - \frac{\sec[(a+p)u]}{2(a+p)}$$

$$292) \int \frac{du}{1-\cos(au)} = -\frac{1}{a} \cot \frac{au}{2}$$

$$293) \int \frac{u du}{1-\cos(au)} = -\frac{u}{a} \cot \frac{au}{2} + \frac{2}{a^2} \ln \sin \frac{au}{2}$$

$$294) \int \frac{du}{1+\cos(au)} = \frac{1}{a} \tan \frac{au}{2}$$

$$295) \int \frac{u du}{1+\cos(au)} = \frac{u}{a} \tan \frac{au}{2} + \frac{2}{a^2} \ln \cos \frac{au}{2}$$

$$296) \int \frac{du}{(1-\cos(au))^2} = -\frac{1}{2a} \cot \frac{au}{2} - \frac{1}{6a} \cot^3 \frac{au}{2}$$

$$297) \int \frac{du}{(1+\cos(au))^2} = \frac{1}{2a} \tan \frac{au}{2} + \frac{1}{6a} \tan^3 \frac{au}{2}$$

Integrales con $\sin(au)$ y $\cos(au)$.

$$298) \int \sin(au) \cos(au) du = \frac{\sin^2(au)}{2a}$$

$$299) \int \sin(pu) \cos(qu) du = -\frac{\cos[(p-q)u]}{2(p-q)} - \frac{\cos[(p+q)u]}{2(p+q)}$$

$$300) \int \sin^n(au) \cos(au) du = \frac{\sin^{n+1}(au)}{(n+1)a}$$

$$301) \int \cos^n(au) \sin(au) du = -\frac{\cos^{n+1}(au)}{(n+1)a}$$

$$302) \int \sin^2(au) \cos^2(au) du = \frac{u}{8} - \frac{\sin 4(au)}{32a}$$

$$303) \int \frac{du}{\sin(au) \cos(au)} = \frac{1}{a} \ln \left[\tan(au) \right]$$

$$304) \int \frac{du}{\sin^2(au) \cos(au)} = \frac{1}{a} \ln \left[\tan \left(\frac{\pi}{4} + \frac{au}{2} \right) \right] - \frac{1}{a \sin(au)}$$

$$305) \int \frac{du}{\sin(au) \cos^2(au)} = \frac{1}{a} \ln \left[\tan \left(\frac{au}{2} \right) \right] + \frac{1}{a \cos(au)}$$

$$306) \int \frac{du}{\sin^2(au) \cos^2(au)} = -\frac{2 \cot(2au)}{a}$$

$$307) \int \frac{\sin^2(au)}{\cos(au)} du = -\frac{\sin(au)}{a} + \frac{1}{a} \ln \left[\tan \left(\frac{au}{2} + \frac{\pi}{4} \right) \right]$$

$$308) \int \frac{\cos^2(au)}{\sin(au)} du = \frac{\cos(au)}{a} + \frac{1}{a} \ln \left[\tan \left(\frac{au}{2} \right) \right]$$

$$309) \int \frac{du}{\sin(au) \pm \cos(au)} = \frac{1}{a\sqrt{2}} \ln \tan \left(\frac{au}{2} \pm \frac{\pi}{8} \right)$$

$$310) \int \frac{\sin(au) du}{\sin(au) \pm \cos(au)} = \frac{x}{2} \mp \frac{1}{2a} \ln [\sin(au) \pm \cos(au)]$$

$$311) \int \frac{\cos(au) du}{\sin(au) \pm \cos(au)} = \pm \frac{x}{2} + \frac{1}{2a} \ln [\sin(au) \pm \cos(au)]$$

Integrales con $\tan(au)$.

$$312) \int \tan(au) du = -\frac{1}{a} \ln \cos(au) = \frac{1}{a} \ln \sec(au)$$

$$313) \int \tan^2(au) du = \frac{\tan(au)}{a} - u$$

$$314) \int \tan^3(au) du = \frac{\tan^2(au)}{2a} + \frac{1}{a} \ln \cos(au)$$

$$315) \int \tan^n(au) du = \frac{\tan^{n-1}(au)}{(n-1)a} - \int \tan^{n-2}(au) du$$

$$316) \int \tan^n(au) \sec^2(au) du = \frac{\tan^{n+1}(au)}{(n+1)a}$$

$$317) \int \frac{\sec^2(au)}{\tan(au)} du = \frac{1}{a} \ln \tan(au)$$

$$318) \int \frac{du}{\tan(au)} = \frac{1}{a} \ln \sin(au)$$

$$319) \int u \tan^2(au) du = \frac{u \tan(au)}{a} + \frac{1}{a^2} \ln \cos(au) - \frac{u^2}{2}$$

Integrales con $\cot(au)$.

$$320) \int \cot(au) du = \frac{1}{a} \ln \sin(au)$$

$$321) \int \cot^2(au) du = -\frac{\cot(au)}{a} - u$$

$$322) \int \cot^3(au) du = -\frac{\cot^2(au)}{2a} - \frac{1}{a} \ln \sin(au)$$

$$323) \int \cot^n(au) \csc^2(au) du = -\frac{\cot^{n+1}(au)}{(n+1)a}$$

$$324) \int \frac{\csc^2(au)}{\cot(au)} du = -\frac{1}{a} \ln \cot(au)$$

$$325) \int \frac{du}{\cot(au)} = -\frac{1}{a} \ln \cos(au)$$

$$326) \int u \cot^2(au) du = -\frac{u \cot(au)}{a} + \frac{1}{a^2} \ln \sin(au) - \frac{u^2}{2}$$

$$327) \int \cot^n(au) du = -\frac{\cot^{n-1}(au)}{(n-1)a} - \int \cot^{n-2}(au) du$$

Integrales con $\sec(au)$.

$$328) \int \sec(au) du = \frac{1}{a} \ln [\sec(au) + \tan(au)] = \frac{1}{a} \ln \tan \left(\frac{ax}{2} + \frac{\pi}{4} \right)$$

$$329) \int \sec^2(au) du = \frac{\tan(au)}{a}$$

$$330) \int \sec^3(au) du = \frac{\sec(au) \tan(au)}{2a} + \frac{1}{2a} \ln [\sec(au) + \tan(au)]$$

$$331) \int \sec^n(au) \tan(au) du = \frac{\sec^n(au)}{na}$$

$$332) \int \frac{du}{\sec(au)} = \frac{\sin(au)}{a}$$

$$333) \int u \sec^2(au) du = \frac{x}{a} \tan(au) + \frac{1}{a^2} \ln \cos(au)$$

$$334) \int \sec^n(au) du = \frac{\sec^{n-2}(au) \tan(au)}{a(n-1)} + \frac{n-2}{n-1} \int \sec^{n-2}(au) du$$

Integrales con $\csc(au)$.

$$335) \int \csc(au) du = \frac{1}{a} \ln [\csc(au) - \cot(au)] = \frac{1}{a} \ln \left[\tan \frac{au}{2} \right]$$

$$336) \int \csc^2(au) du = -\frac{\cot(au)}{a}$$

$$337) \int \csc^3(au) du = -\frac{\csc(au) \cot(au)}{2a} + \frac{1}{2a} \ln \left[\tan \frac{au}{2} \right]$$

$$338) \int \csc^n(au) \cot(au) du = -\frac{\csc^n(au)}{na}$$

$$339) \int \frac{du}{\csc(au)} = -\frac{\cos(au)}{a}$$

$$340) \int u \csc^2(au) du = -\frac{u \cot(au)}{a} + \frac{1}{a^2} \ln [\sin(au)]$$

$$341) \int \csc^n(au) du = -\frac{\csc^{n-2}(au) \cot(au)}{a(n-1)} + \frac{n-2}{n-1} \int \csc^{n-2}(au) du$$

Integrales de Funciones Trigonométricas Inversas.

- 342) $\int \sin^{-1}(u/a) du = u \sin^{-1}(u/a) + \sqrt{a^2 - u^2}$
- 343) $\int u \sin^{-1}(u/a) du = \left(\frac{u^2}{2} - \frac{a^2}{4}\right) \sin^{-1}(u/a) + \frac{u\sqrt{a^2 - u^2}}{4}$
- 344) $\int u^2 \sin^{-1}(u/a) du = \frac{u^3}{3} \sin^{-1}(u/a) + \frac{(u^2 + 2a^2)\sqrt{a^2 - u^2}}{9}$
- 345) $\int \frac{\sin^{-1}(u/a)}{u} du = \frac{u}{a} + \frac{(u/a)^3}{2 \cdot 3 \cdot 3} + \frac{1 \cdot 3 (u/a)^5}{2 \cdot 4 \cdot 5 \cdot 5} + \frac{1 \cdot 3 \cdot 5 (u/a)^7}{2 \cdot 4 \cdot 6 \cdot 7 \cdot 7} + \dots$
- 346) $\int \frac{\sin^{-1}(u/a)}{u^2} du = -\frac{\sin^{-1}(u/a)}{u} - \frac{1}{a} \ln \left(\frac{a + \sqrt{a^2 - u^2}}{u} \right)$
- 347) $\int \left(\sin^{-1} \frac{u}{a} \right)^2 du = u \left(\sin^{-1} \frac{u}{a} \right)^2 - 2u + 2\sqrt{a^2 - u^2} \sin^{-1} \frac{u}{a}$
- 348) $\int \cos^{-1}(u/a) du = u \cos^{-1} \frac{u}{a} - \sqrt{a^2 - u^2}$
- 349) $\int u \cos^{-1}(u/a) du = \left(\frac{u^2}{2} - \frac{a^2}{4}\right) \cos^{-1} \frac{u}{a} - \frac{u\sqrt{a^2 - u^2}}{4}$
- 350) $\int u^2 \cos^{-1}(u/a) du = \frac{u^3}{3} \cos^{-1} \frac{u}{a} - \frac{(u^2 - 2a^2)\sqrt{a^2 - u^2}}{9}$
- 351) $\int \frac{\cos^{-1}(u/a)}{u} du = \frac{\pi}{2} \ln(u) - \int \frac{\sin(u/a)}{u} du$
- 352) $\int \frac{\cos^{-1}(u/a)}{u^2} du = -\frac{\cos^{-1}(u/a)}{u} + \frac{1}{a} \ln \left(\frac{a + \sqrt{a^2 - u^2}}{u} \right)$
- 353) $\int \left(\cos^{-1} \frac{u}{a} \right)^2 du = u \left(\cos^{-1} \frac{u}{a} \right)^2 - 2u - 2\sqrt{a^2 - u^2} \cos^{-1} \frac{u}{a}$
- 354) $\int \tan^{-1}(u/a) du = u \tan^{-1}(u/a) - \frac{a}{2} \ln(u^2 + a^2)$
- 355) $\int u \tan^{-1}(u/a) du = \frac{1}{2} (u^2 + a^2) \tan^{-1}(u/a) - \frac{au}{2}$
- 356) $\int u^2 \tan^{-1}(u/a) du = \frac{u^3}{3} \tan^{-1}(u/a) - \frac{au^2}{6} + \frac{a^3}{6} \ln(u^2 + a^2)$
- 357) $\int \frac{\tan^{-1}(u/a)}{u} du = (u/a) - \frac{(u/a)^3}{3^2} + \frac{(u/a)^5}{5^2} - \frac{(u/a)^7}{7^2} + \dots$
- 358) $\int \frac{\tan^{-1}(u/a)}{u^2} du = -\frac{1}{u} \tan^{-1}(u/a) - \frac{1}{2a} \ln \left(\frac{u^2 + a^2}{u^2} \right)$
- 359) $\int \cot^{-1}(u/a) du = u \cot^{-1}(u/a) + \frac{a}{2} \ln(u^2 + a^2)$
- 360) $\int u \cot^{-1}(u/a) du = \frac{1}{2} (u^2 + a^2) \cot^{-1}(u/a) + \frac{au}{2}$
- 361) $\int u^2 \cot^{-1}(u/a) du = \frac{u^3}{3} \cot^{-1}(u/a) + \frac{au^2}{6} - \frac{a^3}{6} \ln(u^2 + a^2)$
- 362) $\int \frac{\cot^{-1}(u/a)}{u} du = \frac{\pi}{2} \ln u - \int \frac{\tan^{-1}(u/a)}{u} du$
- 363) $\int \frac{\cot^{-1}(u/a)}{u^2} du = -\frac{\cot^{-1}(u/a)}{u} + \frac{1}{2a} \ln \left(\frac{u^2 + a^2}{u^2} \right)$
- 364) $\int u^m \sin^{-1}(u/a) du = \frac{u^{m+1}}{m+1} \sin^{-1}(u/a) - \frac{1}{m+1} \int \frac{u^{m+1}}{\sqrt{a^2 - u^2}} du$
- 365) $\int u^m \cos^{-1}(u/a) du = \frac{u^{m+1}}{m+1} \cos^{-1}(u/a) + \frac{1}{m+1} \int \frac{u^{m+1}}{\sqrt{a^2 - u^2}} du$
- 366) $\int u^m \tan^{-1}(u/a) du = \frac{u^{m+1}}{m+1} \tan^{-1}(u/a) - \frac{a}{m+1} \int \frac{u^{m+1}}{u^2 + a^2} du$
- 367) $\int u^m \cot^{-1}(u/a) du = \frac{u^{m+1}}{m+1} \cot^{-1}(u/a) + \frac{a}{m+1} \int \frac{u^{m+1}}{u^2 + a^2} du$

Integrales con e^{au} .

- 368) $\int e^{au} du = \frac{e^{au}}{a}$
- 369) $\int u e^{au} du = \frac{e^{au}}{a} \left(u - \frac{1}{a} \right)$
- 370) $\int u^2 e^{au} du = \frac{e^{au}}{a} \left(u^2 - \frac{2u}{a} + \frac{2}{a^2} \right)$
- 371) $\int u^n e^{au} du = \frac{u^n e^{au}}{a} - \frac{n}{a} \int u^{n-1} e^{au} du$
 $= \frac{e^{au}}{a} \left(u^n - \frac{nu^{n-1}}{a} + \frac{n(n-1)u^{n-2}}{a^2} + \dots + \frac{(-1)^n n!}{a^n} \right)$
 con $n =$ entero positivo
- 372) $\int \frac{e^{au}}{u} = \ln(u) + \frac{au}{1 \cdot 1!} + \frac{(au)^2}{2 \cdot 2!} + \frac{(au)^3}{3 \cdot 3!} + \dots$
- 373) $\int \frac{e^{au}}{u^n} du = \frac{-e^{au}}{(n-1)u^{n-1}} + \frac{a}{n-1} \int \frac{e^{au}}{u^{n-1}} du$
- 374) $\int \frac{du}{p + qe^{au}} = \frac{u}{p} - \frac{1}{ap} \ln(p + qe^{au})$
- 375) $\int \frac{du}{(p + qe^{au})^2} = \frac{u}{p^2} + \frac{1}{ap(p + qe^{au})} - \frac{1}{ap^2} \ln|p + qe^{au}|$
- 376) $\int \frac{du}{pe^{au} + qe^{-au}} = \begin{cases} \frac{1}{a\sqrt{pq}} \tan^{-1} \left(\sqrt{\frac{p}{q}} e^{au} \right) \\ \frac{1}{2a\sqrt{-pq}} \ln \left(\frac{e^{au} - \sqrt{-q/p}}{e^{au} + \sqrt{-q/p}} \right) \end{cases}$
- 377) $\int e^{au} \sin(bu) du = \frac{e^{au} [a \sin(bu) - b \cos(bu)]}{a^2 + b^2}$
- 378) $\int e^{au} \cos(bu) du = \frac{e^{au} [a \cos(bu) + b \sin(bu)]}{a^2 + b^2}$
- 379) $\int e^{au} \ln u du = \frac{e^{au} \ln u}{a} - \frac{1}{a} \int \frac{e^{au}}{u} du$

Integrales con $\ln(u)$.

- 380) $\int \ln(u) du = u \ln(u) - u$
- 381) $\int [\ln(u)]^2 du = u [\ln(u)]^2 - 2u \ln(u) + 2u$
- 382) $\int [\ln(u)]^n du = u [\ln(u)]^n - n \int [\ln(u)]^{n-1} du$
- 383) $\int u \ln(u) du = \frac{u^2}{2} \left[\ln(u) - \frac{1}{2} \right]$
- 384) $\int u^m \ln u du = \frac{u^{m+1}}{m+1} \left(\ln u - \frac{1}{m+1} \right)$
- 385) $\int \frac{\ln u}{u} du = \frac{1}{2} \ln^2 u$
- 386) $\int \frac{\ln u}{u^2} du = -\frac{\ln u}{u} - \frac{1}{u}$
- 387) $\int \ln^2 u du = u \ln^2 u - 2u \ln u + 2u$
- 388) $\int \frac{\ln^n u du}{u} = \frac{\ln^{n+1} u}{n+1}$
- 389) $\int \frac{du}{u \ln u} = \ln(\ln u)$
- 390) $\int \ln(u^2 + a^2) du = u \ln(u^2 + a^2) - 2u + 2a \arctan \frac{u}{a}$
- 391) $\int \ln(u^2 - a^2) du = u \ln(u^2 - a^2) - 2u + a \ln \left(\frac{u+a}{u-a} \right)$

Integrales con $\sinh(au)$.

- 392) $\int \sinh(au)du = \frac{\cosh(au)}{a}$
- 393) $\int u \sinh(au)du = \frac{u \cosh(au)}{a} - \frac{\sinh(au)}{a^2}$
- 394) $\int u^2 \sinh(au)du = \left(\frac{u^2}{a} + \frac{2}{a^3}\right) \cosh(au) - \frac{2u}{a^2} \sinh(au)$
- 395) $\int \frac{\sinh(au)}{u} du = au + \frac{(au)^3}{3 \cdot 3!} + \frac{(au)^5}{5 \cdot 5!} + \dots$
- 396) $\int \frac{\sinh(au)}{u^2} du = -\frac{\sinh(au)}{u} + a \int \frac{\cosh(au)}{u} du$
- 397) $\int \frac{du}{\sinh(au)} = \frac{1}{a} \ln \left[\tanh \left(\frac{au}{2} \right) \right]$
- 398) $\int \sinh^2(au)du = \frac{\sinh(au) \cosh(au)}{2a} - \frac{u}{2}$
- 399) $\int u \sinh^2(au)du = \frac{u \sinh(2au)}{4a} - \frac{\cosh(2au)}{8a^2} - \frac{u^2}{4}$
- 400) $\int \frac{du}{\sinh^2(au)} = -\frac{\coth(au)}{a}$
- 401) $\int \sinh(au) \sinh(pu)du = \frac{\sinh[(a+p)u]}{2(a+p)} - \frac{\sinh[(a-p)u]}{2(a-p)}$
- 402) $\int u^m \sinh(au)du = \frac{u^m \cosh(au)}{a} - \frac{m}{a} \int u^{m-1} \cosh(au)du$
- 403) $\int \sinh^n(au)du = \frac{\sinh^{n-1}(au) \cosh(au)}{an} - \frac{n-1}{n} \int \sinh^{n-2}(au)du$
- 404) $\int \frac{\sinh(au)}{u^n} du = \frac{-\sinh(au)}{(n-1)u^{n-1}} + \frac{a}{n-1} \int \frac{\cosh(au)}{u^{n-1}} du$
- 405) $\int \frac{du}{\sinh^n(au)} = \frac{-\cosh(au)}{a(n-1) \sinh^{n-1}(au)} - \frac{n-2}{n-1} \int \frac{du}{\sinh^{n-2}(au)}$

Integrales con $\cosh(au)$.

- 406) $\int \cosh(au)du = \frac{\sinh(au)}{a}$
- 407) $\int u \cosh(au)du = \frac{u \sinh(au)}{a} - \frac{\cosh(au)}{a^2}$
- 408) $\int u^2 \cosh(au)du = -\frac{2u \cosh(au)}{a^2} + \left(\frac{u^2}{a} + \frac{2}{a^3}\right) \sinh(au)$
- 409) $\int \frac{\cosh(au)}{u} du = \ln u + \frac{(au)^2}{2 \cdot 2!} + \frac{(au)^4}{4 \cdot 4!} + \frac{(au)^6}{6 \cdot 6!} + \dots$
- 410) $\int \frac{\cosh(au)}{u^2} du = -\frac{\cosh(au)}{u} + a \int \frac{\sinh(au)}{u} du$
- 411) $\int \frac{du}{\cosh(au)} = \frac{2}{a} \tan^{-1} e^{au}$
- 412) $\int \cosh^2(au)du = \frac{u}{2} + \frac{\sinh(au) \cosh(au)}{2a}$
- 413) $\int u \cosh^2(au)du = \frac{u^2}{4} + \frac{u \sinh(2au)}{4a} - \frac{\cosh(2au)}{8a^2}$
- 414) $\int \frac{du}{\cosh^2(au)} = \frac{\tanh(au)}{a}$
- 415) $\int \cosh(au) \cosh(pu)du = \frac{\sinh[(a-pu)]}{2(a-p)} + \frac{\sinh[(a+p)u]}{2(a+p)}$
- 416) $\int u^m \cosh(au)du = \frac{u^m \sinh(au)}{a} - \frac{m}{a} \int u^{m-1} \sinh(au)du$
- 417) $\int \cosh^n(au)du = \frac{\cosh^{n-1}(au) \sinh(au)}{an} + \frac{n-1}{n} \int \cosh^{n-2}(au)du$
- 418) $\int \frac{\cosh(au)}{u^n} du = \frac{-\cosh(au)}{(n-1)u^{n-1}} + \frac{a}{n-1} \int \frac{\sinh(au)}{u^{n-1}} du$
- 419) $\int \frac{du}{\cosh^n(au)} = \frac{\sinh(au)}{a(n-1) \cosh^{n-1}(au)} + \frac{n-2}{n-1} \int \frac{du}{\cosh^{n-2}(au)}$

5. Transformadas de Laplace.....

$f(s)$	$F(t)$
$\frac{k}{s}$	$k \quad \text{con } k = \text{constante}$
$\frac{1}{s^2}$	t
$\frac{1}{s^n}$	$\frac{t^{n-1}}{(n-1)!} \quad \text{con } n = 0, 1, 2, \dots$
$\frac{1}{s^n}$	$\frac{t^{n-1}}{\Gamma(n)} \quad \text{con } n > 0$
$\frac{1}{s-a}$	e^{at}
$\frac{1}{(s-a)^n}$	$\frac{t^{n-1} e^{at}}{(n-1)!} \quad \text{con } n = 0, 1, 2, \dots$
$\frac{1}{(s-a)^n}$	$\frac{t^{n-1} e^{at}}{\Gamma(n)} \quad \text{con } n > 0$
$\frac{1}{s^2 + a^2}$	$\frac{\sin(at)}{a}$

$f(s)$	$F(t)$
$\frac{s}{s^2 + a^2}$	$\frac{\cos(at)}{a}$
$\frac{1}{(s-b)^2 + a^2}$	$\frac{e^{bt} \sin(at)}{a}$
$\frac{s-b}{(s-b)^2 + a^2}$	$e^{bt} \cos(at)$
$\frac{1}{s^2 - a^2}$	$\frac{\sinh(at)}{a}$
$\frac{s}{s^2 - a^2}$	$\cosh(at)$
$\frac{1}{(s-b)^2 - a^2}$	$\frac{e^{bt} \sinh(at)}{a}$
$\frac{s-b}{(s-b)^2 - a^2}$	$e^{bt} \cosh(at)$
$\frac{1}{(s-a)(s-b)}$	$\frac{e^{bt} - e^{at}}{b-a} \quad \text{con } a \neq b$

A en grados	A en radianes	$\text{sen } A$	$\text{cos } A$	$\text{tan } A$	$\text{cot } A$	$\text{sec } A$	$\text{csc } A$
0°	0	0	1	0	∞	1	∞
15°	$\pi/12$	$\frac{1}{4}(\sqrt{6} - \sqrt{2})$	$\frac{1}{4}(\sqrt{6} + \sqrt{2})$	$2 - \sqrt{3}$	$2 + \sqrt{3}$	$\sqrt{6} - \sqrt{2}$	$\sqrt{6} + \sqrt{2}$
30°	$\pi/6$	$\frac{1}{2}$	$\frac{1}{2}\sqrt{3}$	$\frac{1}{3}\sqrt{3}$	$\sqrt{3}$	$\frac{2}{3}\sqrt{3}$	2
45°	$\pi/4$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{2}$	1	1	$\sqrt{2}$	$\sqrt{2}$
60°	$\pi/3$	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{1}{3}\sqrt{3}$	2	$\frac{2}{3}\sqrt{3}$
75°	$5\pi/12$	$\frac{1}{4}(\sqrt{6} + \sqrt{2})$	$\frac{1}{4}(\sqrt{6} - \sqrt{2})$	$2 + \sqrt{3}$	$2 - \sqrt{3}$	$\sqrt{6} + \sqrt{2}$	$\sqrt{6} - \sqrt{2}$
90°	$\pi/2$	1	0	$\pm\infty$	0	$\pm\infty$	1
105°	$7\pi/12$	$\frac{1}{4}(\sqrt{6} + \sqrt{2})$	$-\frac{1}{4}(\sqrt{6} - \sqrt{2})$	$-(2 + \sqrt{3})$	$-(2 - \sqrt{3})$	$-(\sqrt{6} + \sqrt{2})$	$\sqrt{6} - \sqrt{2}$
120°	$2\pi/3$	$\frac{1}{2}\sqrt{3}$	$-\frac{1}{2}$	$-\sqrt{3}$	$-\frac{1}{3}\sqrt{3}$	-2	$\frac{2}{3}\sqrt{3}$
135°	$3\pi/4$	$\frac{1}{2}\sqrt{2}$	$-\frac{1}{2}\sqrt{2}$	-1	-1	$-\sqrt{2}$	$\sqrt{2}$
150°	$5\pi/6$	$\frac{1}{2}$	$-\frac{1}{2}\sqrt{3}$	$-\frac{1}{3}\sqrt{3}$	$-\sqrt{3}$	$-\frac{2}{3}\sqrt{3}$	2
165°	$11\pi/12$	$\frac{1}{4}(\sqrt{6} - \sqrt{2})$	$-\frac{1}{4}(\sqrt{6} + \sqrt{2})$	$-(2 - \sqrt{3})$	$-(2 + \sqrt{3})$	$-(\sqrt{6} - \sqrt{2})$	$\sqrt{6} + \sqrt{2}$
180°	π	0	-1	0	$\pm\infty$	-1	$\pm\infty$
195°	$13\pi/12$	$-\frac{1}{4}(\sqrt{6} - \sqrt{2})$	$-\frac{1}{4}(\sqrt{6} + \sqrt{2})$	$2 - \sqrt{3}$	$2 + \sqrt{3}$	$-(\sqrt{6} - \sqrt{2})$	$-(\sqrt{6} + \sqrt{2})$
210°	$7\pi/6$	$-\frac{1}{2}$	$-\frac{1}{2}\sqrt{3}$	$\frac{1}{3}\sqrt{3}$	$\sqrt{3}$	$-\frac{2}{3}\sqrt{3}$	-2
225°	$5\pi/4$	$-\frac{1}{2}\sqrt{2}$	$-\frac{1}{2}\sqrt{2}$	1	1	$-\sqrt{2}$	$-\sqrt{2}$
240°	$4\pi/3$	$-\frac{1}{2}\sqrt{3}$	$-\frac{1}{2}$	$\sqrt{3}$	$\frac{1}{3}\sqrt{3}$	-2	$-\frac{2}{3}\sqrt{3}$
255°	$17\pi/12$	$-\frac{1}{4}(\sqrt{6} + \sqrt{2})$	$-\frac{1}{4}(\sqrt{6} - \sqrt{2})$	$2 + \sqrt{3}$	$2 - \sqrt{3}$	$-(\sqrt{6} + \sqrt{2})$	$-(\sqrt{6} - \sqrt{2})$
270°	$3\pi/2$	-1	0	$\pm\infty$	0	$\mp\infty$	-1
285°	$19\pi/12$	$-\frac{1}{4}(\sqrt{6} + \sqrt{2})$	$\frac{1}{4}(\sqrt{6} - \sqrt{2})$	$-(2 + \sqrt{3})$	$-(2 - \sqrt{3})$	$\sqrt{6} + \sqrt{2}$	$-(\sqrt{6} - \sqrt{2})$
300°	$5\pi/3$	$-\frac{1}{2}\sqrt{3}$	$\frac{1}{2}$	$-\sqrt{3}$	$-\frac{1}{3}\sqrt{3}$	2	$-\frac{2}{3}\sqrt{3}$
315°	$7\pi/4$	$-\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{2}$	-1	-1	$\sqrt{2}$	$-\sqrt{2}$
330°	$11\pi/6$	$-\frac{1}{2}$	$\frac{1}{2}\sqrt{3}$	$-\frac{1}{3}\sqrt{3}$	$-\sqrt{3}$	$\frac{2}{3}\sqrt{3}$	-2
345°	$23\pi/12$	$-\frac{1}{4}(\sqrt{6} - \sqrt{2})$	$\frac{1}{4}(\sqrt{6} + \sqrt{2})$	$-(2 - \sqrt{3})$	$-(2 + \sqrt{3})$	$\sqrt{6} - \sqrt{2}$	$-(\sqrt{6} + \sqrt{2})$
360°	2π	0	1	0	$\mp\infty$	1	$\mp\infty$